ELIZADE UNIVERSITY

ILARA-MOKIN

FACULTY: BASIC AND APPLIED SCIENCES
DEPARTMENT: MATHEMATICS AND COMPUTER SCIENCE
1st SEMESTER EXAMINATION
2017 / 2018 ACADEMIC SESSION

COURSE CODE: MTH 203

COURSE TITLE: Linear Algebra I

COURSE LEADER: Dr. A. Adesanya

DURATION: $2^1/_2$ Hours

HOD's SIGNATURE

INSTRUCTION:

Candidates should answer any FOUR Questions.

Students are warned that possession of any unauthorized materials in an examination is a serious offence.

Q1. (a) Define the term Vector Space.

(b) Let V be the set of vectors $[2x - 3y, x + 2y, -y, -4x]$ with $x, y \in R^2$.

Addition and scalar multiplication are defined in the same way as on vectors.

Prove that V is a Vector Space.

Q2. (a) Define Vector Subspace.

Determine if the following given set is a subspace of the given vector space.

(i) Let W be the set of all points (x, y) from R^2 in which $x \geq 0$. Is this subspace of R^2

(ii) Is $S = \left\{ \begin{bmatrix} a \\ b \\ 0 \end{bmatrix} : a, b \in R \right\}$ a subspace of R^3 . Justify your claim.

(b) Let $W = \{(x, y, z) \in R^3 | 3x = 2y\}$. Prove that W is a Subspace .

Q3. (a) Differentiate between Linearly dependent set and linearly independent set.

(b) Determine whether or not $\{V_1, V_2, V_3,\}$ is linearly independent , where

$V_1 = (1, 1, 2, 1)$, $V_2 = (0, 2, 1, 1)$ and $V_3 = (3, 1, 2, 0)$.

(c) What do you understand by Linear combination and Linear Span ?

Express V_3 as a linear combination of V_1 and V_2 given $V_1 = (1, 0, 1)$,

$V_2 = (-1, 1, 0)$ and $V_3 = (1, 2, 3)$.

Q4. . (a) Define a linear transformation.

Let $T: R^2 \rightarrow R^1$ be defined by $T[(X_1, X_2)] = X_1^2 + X_2^2$ .Show that T is not linear.

(b) Let T be the linear transformation defined by $T(x, y) = (3x + 5y, 5x - 2y)$

Computer the matrix T in the basis $\{e_1 = (1,3),\ e_2 = (-1,2)\}$.

Hence prove that $[T]_e[V]_e = [T(V)]_e$ where $V = (2, -7)$.

Q5. (a) Define $L: V \rightarrow U$ by $L[X_1, X_2] = [X_1, X_2 - X_1, X_2]$.

Show that L is a linear transformation from $R^2 \rightarrow R^3$

(b) Let $L: R^3 \rightarrow R^4$ be defined by

$L(X_1, X_2, X_3) = (-6X_2 + 2X_3, X_1 - X_2 + X_3, -X_1 + X_2 - 6X_3, 3X_1 - X_2 + 4X_3)$.

Compute $L(e_1)$, $L(e_2)$, $L(e_3)$. Hence find the matrix representation and compute AX, where $X = (X_1, X_2, X_3)$.

Q6. Let $V = R^2$ and $U = R^3$.

$L: V \rightarrow U$ by $L(X_1, X_2) = (X_1 - X_2, X_1, X_2)$

Let $F = \{(1, 1), (-1, 1)\}$ and $G = \{(1, 0, 1), (0, 1, 1), (1, 1, 0)\}$

(a) Find the matrix representation of L using the standard bases in both V and U.

(b) Find the matrix representation of L using the standard basis in V and the basis G in U .

(c) Find the matrix representation of L using the basis F in R^2 and the standard basis in R^3.